A Distributed Differential Space-Time Coding Scheme with Analog Network Coding in Two-Way Relay Networks

Qiang Huo, Lingyang Song, Yonghui Li, and Bingli Jiao

Abstract

In this paper, we consider general two-way relay networks (TWRNs) with two source and N relay nodes. A distributed differential space time coding with analog network coding (DDSTC-ANC) scheme is proposed. A simple blind estimation and a differential signal detector are developed to recover the desired signal at each source. The pairwise error probability (PEP) and block error rate (BLER) of the DDSTC-ANC scheme are analyzed. Exact and simplified PEP expressions are derived. To improve the system performance, the optimum power allocation (OPA) between the source and relay nodes is determined based on the simplified PEP expression. The analytical results are verified through simulations.

Index Terms

Distributed differential space-time coding, two-way relay network, analog network coding.

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I. Introduction

It is well known that cooperative communication improves system robustness and capacity by allowing nodes to cooperate in their transmission to form a virtual antenna array [1]. Compared to one-way relay networks (OWRN), two-way communication is an effective scheme to improve the spectral efficiency by allowing the simultaneous exchange of two-way information flows. In [2], the authors first studied the two-way relay networks (TWRN) and derived its achievable bidirectional rate. The TWRNs have attracted increased interest due to its high spectral efficiency. Various protocols for the TWRNs have been proposed recently [3], [4].

In [4], the conventional network coding scheme was applied to the TWRNs. In this scheme, two source nodes transmit signals to the relay, separately. The relay decodes the received signals, performs binary network coding, and broadcasts network coded symbols back to both source nodes. However, this scheme may cause irreducible error floor due to the detection errors which occur at the relay node. In [3], an amplify and forward based network coding scheme was proposed. In this scheme, both source nodes transmit at the same time so that the relay receives a superimposed signal. The relay amplifies the received signal, and broadcasts it to both source nodes. Each source node subtracts its own contribution and estimates the signal transmitted from the other source node. Analog network coding is particularly useful in wireless networks as the wireless channel acts as a natural implementation of network coding by summing the wireless signals over the air.

Recently, distributed space-time coding for OWRNs was proposed in [5] to achieve spatial diversity. Since OWRNs take place only in a single-direction, to further improve the spectral efficiency of the relay networks, the distributed space-time coding was proposed for TWRNs in [6], [7]. However, most of the existing works on distributed space-time coding in TWRNs consider coherent detection at each receiver with the assumption of available channel-state information (CSI). In some situations, e.g., the fast fading environment, the acquisition of accurate CSI presents great challenge, and training becomes expensive and inefficient while there are a large number of relays in the wireless networks [8]. In this case, differential modulation would be a practical solution because it requires no knowledge of the CSI.

The distributed differential space-time coding was first proposed for OWRNs in [9]. In TWRNs, the signal received at the relay node is a superposition of two symbols sent from two source

nodes. Thus if there is no CSI available at source and relay nodes, it will be very difficult to design distributed differential modulation schemes in TWRNs. The challenge is due to the blind channel estimation from the superimposed signals at the relay and unknown self-interference at each destination. In [10], the authors first extended the distributed differential space-time coding to TWRNs. In order to enable differential encoding and decoding, this scheme starts with a four-stage initialization phase, which is similar to traditional one-way relaying, to transmit the bi-directional reference signals respectively. After initialization, each user then proceeds to the data transmission. Information exchange between two users is done in two time slots. However, the decoding algorithm in [10] is a non-coherent detection scheme where the decoding of current symbol is based on the estimation of the previous symbol. Consequently, when one symbol was decoded incorrectly, it will affect the decoding of consecutive symbols thus leading to serious error propagation. To solve this problem, periodical initialization of the protocol has to be performed to transmit new reference signals for decoding, making the proposed scheme inefficient. Furthermore, no pairwise error probability (PEP) analysis was performed in [10] due to the complexity of the protocol. [8] presented an analog network coding scheme with differential modulation using the amplify-and-forward protocol for bidirectional relay networks. However, this scheme is limited to single relay node, thus cannot be extended to the distributed space-time codes.

Unlike [8]–[10], in this paper, we propose a distributed differential space time coding with analog network coding (DDSTC-ANC) scheme for the TWRNs with multiple relays. In this scheme, two source nodes perform differential modulation, and transmit the differential modulated symbols to all the relay nodes in the first time slot. The signal received at the relay node is a superposition of two transmitted symbols. In the second time slot, the N relay nodes broadcast the processed signals to both source nodes simultaneously. We propose a blind estimation technique which can be used to subtract the self-interference without knowledge of CSI at both relay nodes and two source nodes. A simple differential signal detector is then developed to recover the desired signal at each source. The performance of the proposed differential DDSTC-ANC scheme is analyzed and the PEP and block error rate (BLER) expressions are derived. They show that the proposed differential scheme can achieve the same diversity order as the coherent detection scheme but is about 3dB away compared to the coherent detection scheme due to the differential transmission. To further improve the system performance, the optimum power allocation (OPA)

between the source nodes and the relay nodes is determined based on the provided simplified PEP expression. The analytical results are verified through simulations. Simulation results also show that the proposed differential scheme with OPA yields superior performance improvement over an equal power allocation (EPA) scheme.

The rest of this paper is organized as follows: In Section II, the system model is introduced. Section III presents the proposed DDSTC-ANC scheme. The performance and diversity order of DDSTC-ANC are analyzed in Section IV. In Section V, the OPA for the DDSTC-ANC is presented. Simulation results are provided in Section VI. In Section VII, we draw the main conclusions.

Notation: Matrices and vectors are denoted using capital letters and boldface lowercase letters, respectively. $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ represent conjugate, transpose and conjugate transpose, respectively, for both matrix and vector. For a complex matrix A, $\det A$ and tr$A$ denote the determinant and trace of A, respectively. $\mathbf{I}_m$ is the $m \times m$ identity matrix. diag$\{a_1, \cdots, a_n\}$ stands for an $n \times n$ diagonal matrix whose ith diagonal entry is a_i. $\ln$ represents the natural logarithm, and $||\cdot||$ is the Frobenius norm. $\mathbb{E}$ and $P(\cdot)$ denote the expectation and probability, respectively.

II. System Model

In this paper, we consider a general TWRN with $N+2$ nodes, as shown in Fig. 1, where two source nodes, T_1 and T_2, want to exchange information with each other through N relay nodes. It is assumed that each node in the network is equipped with one single antenna working in the half-duplex mode. We consider a quasi-static fading channel, where the channel remains constant for the duration of a frame and varies independently from one frame to another. Let f_i and g_i denote the complex fading channel coefficients of $T_1 - R_i$ and $T_2 - R_i$, respectively. Furthermore, we assume Rayleigh flat fading channels, i.e., $f_i \sim \mathcal{CN}(0, \sigma_{f_i}^2)$ and $g_i \sim \mathcal{CN}(0, \sigma_{g_i}^2)$, respectively. For analysis tractability, symmetry of the relay nodes is assumed in this paper, i.e., $\sigma_{f_i} = \sigma_f, \forall i$ and $\sigma_{g_i} = \sigma_g, \forall i$.

A general two-time slot TWRN protocol is used, as shown in Fig. 1. In the first time slot, both T_1 and T_2 transmit their messages and the relays $\{R_1, \cdots, R_N\}$ receive a superposition of the signals transmitted from T_1 and T_2. Let $\mathbf{s}(t) = [s_1(t), \cdots, s_T(t)]^T$ and $\mathbf{d}(t) = [d_1(t), \cdots, d_T(t)]^T$ denote the transmitted symbol vectors of T_1 and T_2 at time t, respectively. They are normalized

as $\mathbb{E}\{\mathbf{s}(t)\mathbf{s}(t)^H\} = \mathbb{E}\{\mathbf{d}(t)\mathbf{d}(t)^H\} = \mathbf{I}_T$. The received signal vector at R_i can be written as

$$\mathbf{r}_i(t) = \sqrt{P_1} f_i(t)\, \mathbf{s}(t) + \sqrt{P_2} g_i(t)\, \mathbf{d}(t) + \mathbf{v}_i(t), \tag{1}$$

where P_1 and P_2 denote the transmit power of T_1 and T_2, respectively, and $\mathbf{v}_i(t)$ represents the noise vector at R_i and each noise term follows a zero-mean complex additive white Gaussian distribution, i.e., $\mathbf{v}_i(t) \sim \mathcal{CN}(0, N_0 \mathbf{I}_T)$.

During the second time slot, R_i processes $\mathbf{r}_i(t)$ to generate a space time coded symbol vector $\mathbf{x}_i(t)$. In this paper, we consider the amplify-and-forward protocol in the relay nodes. The transmit signal at the i-th relay is designed to be a linear function of its received signal and its conjugate [11]:

$$\mathbf{x}_i(t) = \beta_i(t) \left(A_i \mathbf{r}_i(t) + B_i \mathbf{r}_i(t)^* \right), \tag{2}$$

where A_i and B_i are two $T \times T$ complex matrices specifically designed for the construction of distributed space-time codings, and $\beta_i(t)$ is the scaling factor at R_i.

In this work, the scaling factor $\beta_i(t)$ in Eq. (2) can be obtained based on the available statistical CSI, which is specifically given by [12]

$$\beta_i(t) = \sqrt{\frac{P_{R_i}}{\sigma_{f_i}^2 P_1 + \sigma_{g_i}^2 P_2 + N_0}}, \tag{3}$$

where P_{R_i} is the transmitted power of R_i. Since we assume $P_{R_i} = P_R$, we have $\beta_i(t) = \beta, \forall i$. For simplicity, in this paper, we only design the system that either A_i is unitary, $B_i = \mathbf{0}_T$ (case I) or B_i is unitary, $A_i = \mathbf{0}_T$ (case II). Thus, case I means that the i-th column of the code matrix ($S(t)$ and $D(t)$ in Eq. (6)) contains only the transmitted symbols, and case II means that the i-th column of the code matrix contains the linear combinations of the conjugate of the transmitted symbols only. Further more, we assume that $T = N$, i.e., the number of symbols in a space-time block code is equal to the number of relay nodes. We further define

$$\begin{cases} O_i \triangleq A_i,\ \hat{f}_i \triangleq f_i,\ \hat{g}_i \triangleq g_i,\ \hat{\mathbf{v}}_i(t) \triangleq \mathbf{v}_i(t), \\ \quad \hat{\mathbf{s}}_i(t) \triangleq \mathbf{s}(t),\ \hat{\mathbf{d}}_i(t) \triangleq \mathbf{d}(t), \quad \text{if } B_i = \mathbf{0}_T, \\ O_i \triangleq B_i,\ \hat{f}_i \triangleq f_i^*,\ \hat{g}_i \triangleq g_i^*,\ \hat{\mathbf{v}}_i(t) \triangleq \mathbf{v}_i(t)^*, \\ \quad \hat{\mathbf{s}}_i(t) \triangleq \mathbf{s}(t)^*,\ \hat{\mathbf{d}}_i(t) \triangleq \mathbf{d}(t)^*,\ \text{if } A_i = \mathbf{0}_T. \end{cases} \tag{4}$$

Then the relay node R_i broadcasts the coded symbol vector $\mathbf{x}_i(t)$ back to both source nodes. Since T_1 and T_2 are mathematically symmetrical, for simplicity, in the following, we only discuss the decoding and the analysis for the signals received by T_2 [13]. The received signal vectors at T_2 is given by

$$\mathbf{y}_2(t) = \sum_{i=1}^{N} g_i(t)\mathbf{x}_i(t) + \mathbf{w}_2(t), \tag{5}$$

where $\mathbf{w}_2(t)$ denotes the independent and identically distributed (i.i.d) additive white Gaussian noise (AWGN) vectors at T_2 and we have $\mathbf{w}_2(t) \sim \mathcal{CN}(0, N_0\mathbf{I}_T)$.

The received signal at T_2 can then be rewritten as:

$$\mathbf{y}_2(t) = \sqrt{P_1}\, S(t)\mathbf{h}_{12}(t) + \sqrt{P_2}\, D(t)\mathbf{h}_{22}(t) + \mathbf{n}_2(t), \tag{6}$$

where

$$\begin{cases} S(t) = \left[O_1\hat{\mathbf{s}}_1(t), \cdots, O_N\hat{\mathbf{s}}_N(t)\right], \\ D(t) = [O_1\hat{\mathbf{d}}_1(t), \cdots, O_N\hat{\mathbf{d}}_N(t)], \\ \mathbf{h}_{12}(t) = [\beta_1(t)\hat{f}_1(t)g_1(t),\ \cdots, \beta_N(t)\hat{f}_N(t)g_N(t)]^T, \\ \mathbf{h}_{22}(t) = [\beta_1(t)\hat{g}_1(t)g_1(t), \cdots, \beta_N(t)\hat{g}_N(t)g_N(t)]^T, \\ \mathbf{n}_2(t) = \sum_{i=1}^{N} \beta_i(t)\ g_i(t)O_i\hat{\mathbf{v}}_i(t) + \mathbf{w}_2(t). \end{cases} \tag{7}$$

It is easy to prove that $\mathbb{E}\{\mathbf{n}_2(t)\mathbf{n}_2(t)^H\} = \sigma^2_{\mathbf{n}_2}(t)\mathbf{I}_N$, and $\sigma^2_{\mathbf{n}_2}(t) = (\sum_{i=1}^{N} |\beta_i(t)|^2\ |g_i(t)|^2 + 1)N_0$.

III. Distributed Differential Space-Time Coding For TWRNs

In this section, we propose a distributed differential scheme. First, we blindly estimate channel $\mathbf{h}_{22}(t)$ defined in Eq. (6), which can be used to subtract the self-interference. Then, a simple differential signal detector is developed to recover the desired signal at source T_2.

In the proposed DDSTC-ANC, T_1 encodes a message at time t into an $N \times N$ unitary matrix $U(t)$, which is then differentially encoded as $\mathbf{s}(t) = U(t) \cdot \mathbf{s}(t-1)$, where $\mathbf{s}(t-1)$ is the signal transmitted by T_1 at time $t-1$. Similarly, T_2 differentially encodes a message at time t into an $N \times N$ unitary matrix $V(t)$, which is then differentially encoded as $\mathbf{d}(t) = V(t) \cdot \mathbf{d}(t-1)$.

For the first block, we can transmit a known vector to both source nodes that satisfies $\mathbb{E}\{\mathbf{s}(t)^H\mathbf{s}(t)\} = \mathbb{E}\{\mathbf{d}(t)^H\mathbf{d}(t)\} = N$, for example, $[1\ 1\ \cdots\ 1]^T$ or $[\sqrt{N}\ 0\ \cdots\ 0]^T$. Similar

to the differential space-time coding for multiple-antenna systems, having $U(t)$ and $V(t)$ unitary preserves the transmit power.

For simplicity, we define $\hat{U}_i(t) \triangleq U(t)$ if $B_i = \mathbf{0}_T$, and $\hat{U}_i(t) \triangleq U(t)^*$ if $A_i = \mathbf{0}_T$. In the distributed differential scheme, the codes $U(t)$ and $V(t)$ should commute with the relay matrices [9], i.e., [1]

$$O_i\hat{U}_i(t) = U(t)O_i,$$

or equivalently,

$$\begin{cases} A_iU(t) = U(t)A_i, & \text{if } B_i = \mathbf{0}_T, \\ B_iU^*(t) = U(t)B_i, & \text{if } A_i = \mathbf{0}_T. \end{cases} \tag{8}$$

Hence, $S(t)$ can be rewritten as

$$\begin{aligned} S(t) &= \Big[O_1\hat{U}_1(t)\hat{\mathbf{s}}_1(t-1), \cdots, O_N\hat{U}_N(t)\hat{\mathbf{s}}_N(t-1)\Big] \\ &= U(t) \cdot (O_1\hat{\mathbf{s}}_1(t-1), \cdots, O_N\hat{\mathbf{s}}_N(t-1)) \\ &= U(t) \cdot S(t-1). \end{aligned} \tag{9}$$

Similarly, we have $D(t) = V(t) \cdot D(t-1)$.

The distributed differential space-time codes (STC) for TWRNs should be designed to satisfy Eq. (8). The design and choice of appropriate codes is beyond the scope of this work, here, we only briefly introduce some existing STCs that can be used in TWRNs. For the TWRNs with two relays, we can use Alamouti code [18], which has full diversity and linear decoding complexity. Square real orthogonal codes (SORC), which have full diversity and linear decoding complexity, were proposed in [9] for two, four and eight antennas systems.

Theorem 1: If the relay matrices have the property: tr$\{O_jO_i^H\} = N$ for $i = j$, tr$\{O_jO_i^H\} = 0$ for $i \neq j$, we have

$$\mathbb{E}\{D(t)^H\mathbf{y}_2(t)\} = \sqrt{P_2}N\,\mathbf{h}_{22}(t), \tag{10}$$

and $\mathbf{h}_{22}(t)$ can be approximated as

$$\mathbf{h}_{22}(t) \approx \frac{1}{NL}\frac{1}{\sqrt{P_2}}\sum_{l=1}^{L} D(t-l)^H\mathbf{y}_2(t-l), \tag{11}$$

[1]More properties about the differential space-time coding can be found in [14]–[17].

where L denotes the number of STC symbols in a frame.

Proof: It can be proved by direct matrix multiplication and expectation. Due to the limited space, we omit the details. ■

We note that since receiver T_2 knows the symbols $\mathbf{d}(t)$ sent by itself, using the blindly estimated channel $\mathbf{h}_{22}(t)$, we can subtract the self-interference at T_2 without using pilot symbols at the beginning. Although we can blindly estimate channel $\mathbf{h}_{22}(t)$, T_2 doesn't have any CSI of $\mathbf{h}_{12}(t)$. Then based on the above theorem, a simple differential signal detector is developed to recover the desired signal $\mathbf{s}(t)$ at source T_2. In the later performance analysis section, we assume that $\mathbf{h}_{22}(t)$ is perfectly cancelled. Most of papers on distributed STCs for TWRNs also assume perfect self-interference cancellation, such as [7], [19] for coherent systems and [8], [13] for differential systems. However, in practice, the estimation error will introduce some performance degradation which depends on estimation accuracy of $\mathbf{h}_{12}(t)$. The estimated $\mathbf{h}_{22}(t)$ is used in simulations in this paper. In the simulation section, we have simulated the proposed scheme using the estimated $\mathbf{h}_{22}(t)$ and the results show that the performance loss due to the $\mathbf{h}_{22}(t)$ estimation error is negligible.

By using Eq. (9), Eq. (11) and the assumption of $\mathbf{h}_{12}(t) = \mathbf{h}_{12}(t-1)$, we have

$$\begin{aligned}\tilde{\mathbf{y}}_2(t) &= \mathbf{y}_2(t) - \sqrt{P_2}\,D(t)\mathbf{h}_{22}(t)\\ &= \sqrt{P_1}\,S(t)\mathbf{h}_{12}(t) + \mathbf{n}_2(t)\\ &= U(t)\tilde{\mathbf{y}}_2(t-1) + \tilde{\mathbf{n}}_2(t),\end{aligned} \tag{12}$$

where $\tilde{\mathbf{n}}_2(t) = \mathbf{n}_2(t) - U(t)\mathbf{n}_2(t-1)$. Note that $\mathbb{E}\{U(t)U(t)^H\} = \mathbf{I}_N$, and $\mathbf{n}_2(t)$ and $\mathbf{n}_2(t-1)$ are independent complex Gaussian random vectors with zero mean and covariance $\sigma^2_{\mathbf{n}_2}(t)$. We have $\mathbb{E}\{\tilde{\mathbf{n}}_2(t)\tilde{\mathbf{n}}_2(t)^H\} = \sigma^2_{\tilde{\mathbf{n}}_2}(t)\mathbf{I}_T$, where $\sigma^2_{\tilde{\mathbf{n}}_2}(t) = 2(\sum_{i=1}^N |\beta_i(t)|^2|g_i(t)|^2 + 1)N_0$. Thus, $\tilde{\mathbf{n}}_2(t)$ is a Gaussian random vector with zero mean and covariance $\sigma^2_{\tilde{\mathbf{n}}_2}(t)$.

Hence, the least square (LS) decoder can be performed to recover the transmitted signal

$$\arg\min_{U_k(t)} \|\tilde{\mathbf{y}}_2(t) - U_k(t)\tilde{\mathbf{y}}_2(t-1)\|. \tag{13}$$

IV. Pairwise Error Probability and Block Error Rate Analysis

In this section, we derive the PEP and the BLER of the proposed DDSTC-ANC scheme. Asymptotic diversity order is also analyzed in this section.

A. Pairwise Error Probability

For simplicity, we define $U_{\Delta,kj}(t) = U_k(t) - U_j(t)$ and $S_{\Delta,kj}(t) = S_k(t) - S_j(t)$. The PEP of mistaking the k-th STC block by the j-th STC block can be evaluated by averaging the conditional PEP over the channel statistics, i.e., f_i, g_i, and we have[2] [20]

$$P_{kj}^d(\gamma) = \mathbb{E}_{f_i,g_i}\left[Q\left(\sqrt{\frac{\|U_{\Delta,kj}(t)\tilde{\mathbf{y}}_2(t-1)\|^2}{2\sigma^2_{\tilde{\mathbf{n}}_2}(t)}}\right)\right], \tag{14}$$

where $\gamma = P/N_0$ is signal-to-noise ratio (SNR), P is the total power in the TWRN and $Q(x) = 1/\sqrt{2\pi}\int_x^\infty \exp(-t^2/2)\mathrm{d}t$ is the Gaussian Q-function. Since it is very difficult to analyse $\tilde{\mathbf{y}}_2(t-1)$ directly, we approximate it using Eq. (12) as $\tilde{\mathbf{y}}_2(t) \approx \sqrt{P_1}S(t)\mathbf{h}_{12}(t)$. This approximation is particularly accurate at high SNR. Then based on Eq. (9), we have $S_{\Delta,ij}(t) = U_{\Delta,ij}(t)S(t-1)$. We further assume $\mathbf{h}_{12}(t-1) \approx \mathbf{h}_{12}(t)$. Then Eq. (14) can be further simplified as

$$P_{kj}^d(\gamma) \approx \mathbb{E}_{f_i,g_i}\, Q\left(\sqrt{\frac{P_1\|S_{\Delta,kj}(t)\mathbf{h}_{12}(t)\|^2}{2\sigma^2_{\tilde{\mathbf{n}}_2}(t)}}\right). \tag{15}$$

Similarly, the PEP for the coherent scheme can be derived as

$$P_{kj}^c(\gamma) = \mathbb{E}_{f_i,g_i}\, Q\left(\sqrt{\frac{P_1\|S_{\Delta,kj}(t)\mathbf{h}_{12}(t)\|^2}{2\sigma^2_{\mathbf{n}_2}(t)}}\right). \tag{16}$$

Since $\sigma^2_{\tilde{\mathbf{n}}_2}(t) = 2\sigma^2_{\mathbf{n}_2}(t)$, the distributed differential scheme in TWRN is supposed to have $3\mathrm{dB}$ loss in coding gain compared to distributed coherent scheme.

Before deriving the PEP, we first define $\mathbf{h}_{12}(t) = \beta G(t)\hat{\mathbf{f}}(t)$, where $\hat{\mathbf{f}}(t) = [\hat{f}_1(t), \cdots, \hat{f}_N(t)]^T$ and $G(t) = diag\{g_1(t), \cdots, g_N(t)\}$. Then we have the following lemmas.

Lemma 2: The probability density function (PDF) of $\hat{\mathbf{f}}(t)$ can be derived as

$$p\left(\hat{\mathbf{f}}(t)\right) = \frac{1}{\pi^N\sigma_f^{2N}}\exp\left(-\frac{\hat{\mathbf{f}}(t)^H\hat{\mathbf{f}}(t)}{\sigma_f^2}\right). \tag{17}$$

Proof: Since $f_i(t) \sim \mathcal{CN}(0, \sigma_f^2)$, we can prove that $f_i^*(t) \sim \mathcal{CN}(0, \sigma_f^2)$. Hence, $\hat{f}_i(t) \sim \mathcal{CN}(0, \sigma_f^2)$. Note that $\hat{f}_1(t), \cdots, \hat{f}_N(t)$ are independent, we can easily derive Eq. (17). ■

Lemma 3: B represents an $n \times n$ Hermitian matrix (i.e., $B^H = B$), and $\mathbf{x}$ is an $n \times 1$ complex vector. We have

$$\int_{\mathcal{C}^n} \exp\left(-\mathbf{x}^H B\mathbf{x}\right) \mathrm{d}\mathbf{x} = \pi^n \det{}^{-1}(B). \tag{18}$$

[2]The superscript "d" denotes differential scheme and "c" represents coherent scheme.

Proof: Please see [21]. ■

Note that the canonical representation of Gaussian Q-function is in the form of a semi-infinite integral, which makes analysis very difficult. Here, we use an alternative representation of the Gaussian Q-function from [22, Eq. (4.2)] as $Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{x^2}{2sin^2\theta}\right) \mathrm{d}\theta$. Then, by doing some manipulations, we have

$$\begin{aligned} P_{kj}^d(\gamma) &= \mathbb{E}_{f_i,g_i}\, \frac{1}{\pi}\int_0^{\pi/2} \exp\left[-\frac{\hat{\mathbf{f}}(t)^H K(t)\hat{\mathbf{f}}(t)}{2\sin^2\theta}\right] \mathrm{d}\theta \\ &= \frac{1}{\pi}\int_0^{\pi/2} \mathbb{E}_{g_i}\ \left[\det(\mathbf{I}+K'(\theta,t)\right]^{-1} \mathrm{d}\theta \\ &= \frac{1}{\pi}\int_0^{\pi/2} \mathbb{E}_{g_i}\ \left[\prod_{i=1}^{N}\left(1+l(\theta,t)\lambda_i|g_i(t)|^2\right)\right]^{-1} \mathrm{d}\theta, \end{aligned} \tag{19}$$

where $K(t) = \frac{P_1|\beta|^2 G(t)^H S_{\Delta,kj}(t)^H S_{\Delta,kj}(t)G(t)}{4(\sum_{i=1}^N |\beta|^2|g_i(t)|^2+1)N_0}$, $K'(\theta,t) = \frac{\sigma_f^2 K(t)}{2\sin^2\theta}$, $l(\theta,t) = \frac{P_1|\beta|^2\sigma_f^2}{8(\sum_{i=1}^N |\beta|^2|g_i(t)|^2+1)N_0\sin^2\theta}$ and λ_i, $i \in \{1,\cdots,N\}$, denotes the singular value of $S_{\Delta,kj}(t)^H S_{\Delta,kj}(t)$. The second step of the equation is based on the Lemma 2 and Lemma 3.

Note that the mean of $|g_i(t)|^2$ is σ_g^2. It is reasonable to approximate the term $\sum_{i=1}^N |g_i(t)|^2$ in $l(\theta,t)$, by $\sum_{i=1}^N |g_i(t)|^2 \approx N\sigma_g^2$, especially for large N (by the law of large numbers) [5], [12]. Hence,

$$l(\theta,t) \approx l'(\theta) = \frac{P_1|\beta|^2\sigma_f^2}{8(N|\beta|^2\sigma_g^2+1)N_0\sin^2\theta}. \tag{20}$$

Let $|g_i(t)|^2 = \gamma_i(t)$. Since $g_i \sim \mathcal{CN}(0,\sigma_g^2)$, the PDF of $\gamma_i(t)$ can be obtained as $p\left(\gamma_i(t)\right) = \frac{1}{\sigma_g^2}\exp\left(-\frac{\gamma_i(t)}{\sigma_g^2}\right)$. Hence, after doing some manipulations, the MGF-based PEP expression is derived as

$$\begin{aligned} &P_{kj}^d(\gamma) \\ &\approx \frac{1}{\pi}\int_0^{\pi/2} \mathbb{E}_{g_i}\ \left[\prod_{i=1}^{N}\left(1+l'(\theta)\lambda_i|g_i(t)|^2\right)\right]^{-1} \mathrm{d}\theta \\ &= \frac{1}{\pi}\int_0^{\pi/2} \prod_{i=1}^{N}\left[-(\frac{\sin^2\theta}{M_i})\exp\left(\frac{\sin^2\theta}{M_i}\right)\mathbf{Ei}\left(-\frac{\sin^2\theta}{M_i}\right)\right]\mathrm{d}\theta, \end{aligned} \tag{21}$$

where $M_i = \frac{P_1|\beta|^2\sigma_f^2\sigma_g^2}{8(N|\beta|^2\sigma_g^2+1)N_0}\lambda_i$ and $\mathbf{Ei}\,(x) = \int_{-\infty}^{x}\frac{e^t}{t}\mathrm{d}t$, for $x < 0$, is the exponential integral function [23, 8.211.1].

Next let us derive the simplified PEP expression at high SNR. Note that [23, 8.214.1] $\mathbf{Ei}\,(x) = \mathbf{C} + \ln(-x) + \sum_{k=1}^{\infty}\frac{x^k}{k\cdot k!},\ [x<0]$, where $\mathbf{C}$ is Eulers constant and $\mathbf{C} \approx 0.577$ [23, 9.73]. When

x tends to 0, the exponential integral function can be approximated as $\mathbf{Ei}\,(x) \approx \ln(-x)$, for $x < 0$. At high SNR, we have $\exp\left(\frac{\sin^2\theta}{M_i}\right) \approx 1$ and using the approximation for the exponential integral function, we have

$$\begin{aligned} &P_{kj}^d(\gamma) \\ &\approx \frac{1}{\pi}\int_0^{\pi/2}\prod_{i=1}^{N}\left[-(\frac{\sin^2\theta}{M_i})\times\left(2\ln\left(\sin\theta\right)+\ln\left(\frac{1}{M_i}\right)\right)\right]\mathrm{d}\theta. \end{aligned} \tag{22}$$

Note that $\int_0^{\pi/2}\ln\sin x\,\mathrm{d}x = -\frac{\pi}{2}\ln 2$ [23, 4.224.3]. Hence the $\ln(\sin\theta)$ can be ignored, especially at high SNR. Using [23, 3.621.3], we have $\int_0^{\pi/2}\sin^{2m}x\mathrm{d}x = \frac{(2m-1)!!}{(2m)!!}\frac{\pi}{2}$. The PEP can be further simplified as

$$P_{kj}^d(\gamma) \lessapprox \frac{1}{2}\frac{(2N-1)!!}{(2N)!!}\prod_{i=1}^{N}\left[(\frac{1}{M_i})\ln\left(M_i\right)\right]. \tag{23}$$

Finally, we derive the well-known Chernoff-bound-based PEP expression. From Eq. (19), setting $\theta = \pi/2$, and doing some manipulations, the Chernoff-bound-based PEP expression is given as

$$\begin{aligned} P_{kj}^d(\gamma) &\leqslant \frac{1}{2}\,\mathbb{E}_{f_i,g_i}\ \exp\left[-\frac{\hat{\mathbf{f}}(t)^H K(t)\hat{\mathbf{f}}(t)}{2}\right] \\ &= \frac{1}{2}\prod_{i=1}^{N}\left[(\frac{1}{M_i})\ln\left(M_i\right)\right]. \end{aligned} \tag{24}$$

The average BLER can be obtained based on the well-known union bound as

$$P_{BLER}^d(\gamma) \leqslant \sum_{U_k\in\mathcal{U}}\ \sum_{j,\,j\neq k} Pr(U_k)P_{kj}^d(\gamma). \tag{25}$$

B. Diversity Order

In this subsection, we analyze the asymptotic diversity order of the proposed DDSTC-ANC scheme. Firstly, we define the total transmission power is $N\cdot P$. Note that $N\cdot P = N\cdot P_1 + N\cdot P_2 + N^2\cdot P_{Ri}$, $P_1 = \alpha_1 P$, and $P_2 = \alpha_2 P$. Denote the SNR $\gamma = P/N_0$. Then, we rewrite M_i at high SNR as $M_i = C\lambda_i\,\gamma$, where $C = \frac{\alpha_1\frac{(1-\alpha_1-\alpha_2)}{N}\sigma_f^2\sigma_g^2}{8((1-\alpha_1-\alpha_2)\sigma_g^2+\alpha_1\sigma_f^2+\alpha_2\sigma_g^2+1/\gamma)} \approx \frac{\alpha_1\frac{(1-\alpha_1-\alpha_2)}{N}\sigma_f^2\sigma_g^2}{8((1-\alpha_1-\alpha_2)\sigma_g^2+\alpha_1\sigma_f^2+\alpha_2\sigma_g^2)}$. Thus, the simplified PEP at high SNR can be rewritten as

$$\begin{aligned} P_{kj}^d(\gamma) &\approx \frac{1}{2}\frac{(2N-1)!!}{(2N)!!}\frac{1}{\prod_{i=1}^{N}C\lambda_i}\gamma^{-N}\times\prod_{i=1}^{N}\left(\ln(C\lambda_i)+\ln(\gamma)\right) \\ &\approx \left(C'\,\gamma\right)^{-N}\left[\ln(\gamma)\right]^N, \end{aligned} \tag{26}$$

where $C' = \left(\frac{1}{2}\frac{(2N-1)!!}{(2N)!!}\frac{1}{\prod_{i=1}^{N}C\lambda_i}\right)^{-\frac{1}{N}}$. When $S_{\Delta,kj}(t)S_{\Delta,kj}(t)^H$ is full rank, the diversity can be obtained as [24]

$$d = \lim_{\gamma\to\infty} -\frac{\log(P^d_{k,j}(\gamma))}{\log(\gamma)} = N\left(1-\frac{\log\log(\gamma)}{\log(\gamma)}\right). \tag{27}$$

Thus, the diversity of the proposed DDSTC-ANC scheme for TWRNs is $N(1-\log\log(\gamma)/\log(\gamma))$.

V. Optimum Power Allocation

In this section, we derive the OPA between the source nodes and the relay nodes that minimizes the total PEP in the TWRNs. Because the MGF-based PEP expression is very hard to analyze and gives little insight, we use the simplified PEP expression to derive the OPA. Here, we consider the total PEP in the TWRNs, and denote the PEP in T_1 and T_2 as $P^{d,1}_{ij}(\gamma)$ and $P^{d,2}_{ij}(\gamma)$, respectively. C in Subsection IV-B is rewritten as C_{T_1} and C_{T_2} for T_1 and T_2, respectively. Hence, we have

$$\begin{aligned}P^{d,1}_{ij}(\gamma)+P^{d,2}_{ij}(\gamma) &\approx \frac{1}{2}\frac{(2N-1)!!}{(2N)!!}\frac{1}{\prod_{i=1}^{N}\lambda_i}\\ &\times\left(C_{T_1}^{-N}+C_{T_2}^{-N}\right)\gamma^{-N}\left[\ln(\gamma)\right]^N,\end{aligned} \tag{28}$$

where $C_{T_1} \approx \frac{\alpha_2\frac{(1-\alpha_1-\alpha_2)}{N}\sigma_f^2\sigma_g^2}{8((1-\alpha_1-\alpha_2)\sigma_f^2+\alpha_1\sigma_f^2+\alpha_2\sigma_g^2)}$ and $C_{T_2} \approx \frac{\alpha_1\frac{(1-\alpha_1-\alpha_2)}{N}\sigma_f^2\sigma_g^2}{8((1-\alpha_1-\alpha_2)\sigma_g^2+\alpha_1\sigma_f^2+\alpha_2\sigma_g^2)}$. It is obvious that to minimize the PEP at high SNR, we should minimize the $C_{T_1}^{-N}+C_{T_2}^{-N}$ in Eq. (28) .i.e.,

$$\min_{\alpha_1,\alpha_2}\{C_{T_1}^{-N}+C_{T_2}^{-N}\}, \qquad \text{s.t.}\begin{cases}\alpha_1\geqslant 0,\ \alpha_2\geqslant 0,\\ \alpha_1+\alpha_2\leqslant 1.\end{cases} \tag{29}$$

As a special case, when $\sigma_f^2=\sigma_g^2=\sigma^2$, we have $\alpha_1=\alpha_2=\alpha$. Therefor,

$$C_{T_1}=C_{T_2}=\frac{2\alpha(1-2\alpha)\sigma^2}{16N}\leq\frac{\sigma^2}{64N}, \tag{30}$$

with equality when $\alpha=\frac{1}{4}$, or equivalently, $P_1=P_2=\frac{P}{4}$ and $P_{R_i}=\frac{P}{2N}$. Thus, the OPA is such that the source nodes use half the total power and the relay nodes share the other half. *We should emphasize* that this power allocation only works for the TWRNs, in which all channels are assumed to be i.i.d. Rayleigh and no path-loss is considered. It is obvious that it may not be optimal when the path-loss effect is considered in the TWRNs.

As the expression in Eq. (29) is complicated, it is difficult to derive the closed-form solution for OPA when $\sigma_f^2\neq\sigma_g^2$. Here, we use numerical method, such as the nonlinear optimization

method, to obtain the optimal solution. In Section VI, it is interesting to find that when $\sigma_f^2 \neq \sigma_g^2$, $\alpha_1 + \alpha_2 = 0.5$ still holds for the simulated scenarios, which means the source nodes still share half the total power.

VI. Simulations

In this section, we provide simulation results for the proposed DDSTC-ANC scheme. Simulations are performed with PSK modulation and a frame size of 100 symbols over a quasi-static Rayleigh fading channels without specific mention. The estimated $\mathbf{h}_{11}(t)$ and $\mathbf{h}_{22}(t)$ are used in simulations. For comparison, we also present simulations over a GSM channel model with a symbol sampling period of $T_s = 3.693\mu s$ and a maximum Doppler shift of 75 Hz [10]. This ensures a slowly changing channel and allows the assumption of a constant channel over two consecutive time blocks. Without specific mention, we assume that $\sigma_f^2 = \sigma_g^2 = 1$ and the source nodes uses half the total power and the relay nodes share the other half, i.e., $P_1 = \frac{1}{4}P$, $P_2 = \frac{1}{4}P$ and $P_{R_i} = \frac{1}{2N}P$.

From Fig. 2, we present the simulated BLER performance for the proposed DDSTC-ANC schemes using Alamouti for TWRNs. The performance of the corresponding coherent detection is plotted as well for better comparison. It shows that the differential scheme suffers about 3-dB performance loss compared with the corresponding coherent scheme, which has been validated in Subsection IV-A. Fig. 2 also compares the simulated BLER performance for our proposed DDSTC-ANC and the differential scheme in [10]. It can be observed that our proposed scheme is superior to (about 2-dB) the detector in [10]. The main reason is that the differential detection approach employed in [10] was based on the estimation of the previous symbol. Consequently, when one symbol was decoded incorrectly, it will affect the decoding of the consecutive symbols thus leading to serious error propagation. Comparatively, the information about the estimation of the previous symbol is not required in our proposed differential detection and is, thus, able to prevent the error propagation.

In Fig. 3, we include the Genie-aided results by assuming that each source node can perfectly remove its own information from the received signal. It can be noted from the results that the proposed differential detection scheme introduces negligible performance loss compared to the genie-aided scheme. We also compare the BLER performance of the differential scheme over a GSM channel (a practical channel) and a quasi-static Rayleigh fading channel. From the figure,

it can be observed that there is almost no performance loss in a GSM channel compared to the quasi-static Rayleigh fading channel which clearly justifies the robustness of the proposed differential scheme in slow fading channels. It also indicates that the effect of non-constant channel on proposed scheme can be ignored which validate our assumption of quasi-static fading channel model.

In Fig. 4, we show the optimum power allocation scheme of the DDSTC-ANC scheme. It can be seen that more power should be allocated to P_1 when the channels from relay nodes to T_2 are better than the channels from relay nodes to T_1. It is interesting to find that when $\sigma_f^2 \neq \sigma_g^2$, the sources still share half the total power for the optimal power allocation.

In Fig. 5, we examine the BLER performance of the proposed scheme with power allocation for the system with four relay nodes. The SORC is used at relays and signal is modulated from a BPSK constellation. We also take into account the relay's location as: case 1 (the *symmetric* case), where relays are placed halfway between the source nodes, i.e., T_1, T_2, and $\sigma_f^2 = 1$ and $\sigma_g^2 = 1$; and case 2 (the *asymmetric* case), where relays are close to the source node T_2, and $\sigma_f^2 = 1$ and $\sigma_g^2 = 10$. It can be observed from Fig. 5 that the BLER performance of the proposed scheme with power allocation can provide considerable performance gain in comparison with the equal power allocation (EPA) scheme, i.e., $P_1 = P_2 = P_{R_i} = \frac{P}{N+2}$.

VII. Conclusion

In this paper, we have proposed a DDSTC-ANC scheme for TWRNs with multiple relays. A simple differential signal detector was developed to recover the desired signal at each source. The performance of the proposed DDSTC-ANC scheme was analyzed and the OPA was presented to improve the system performance. Analytical results have been verified through Monte-Carlo simulations.

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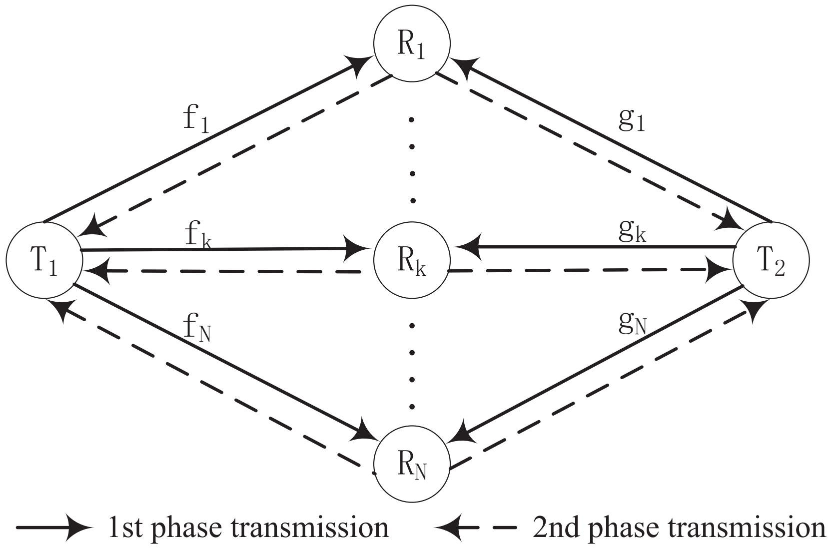

Fig. 1. Block diagram of the two-hop TWRN.

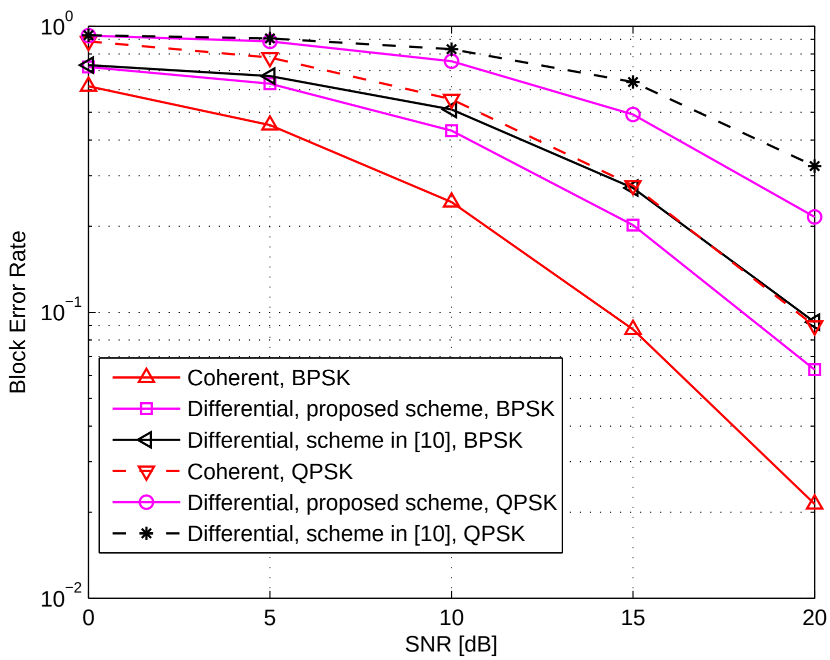

Fig. 2. Simulated BLER performance using Alamouti codes (2 relays).

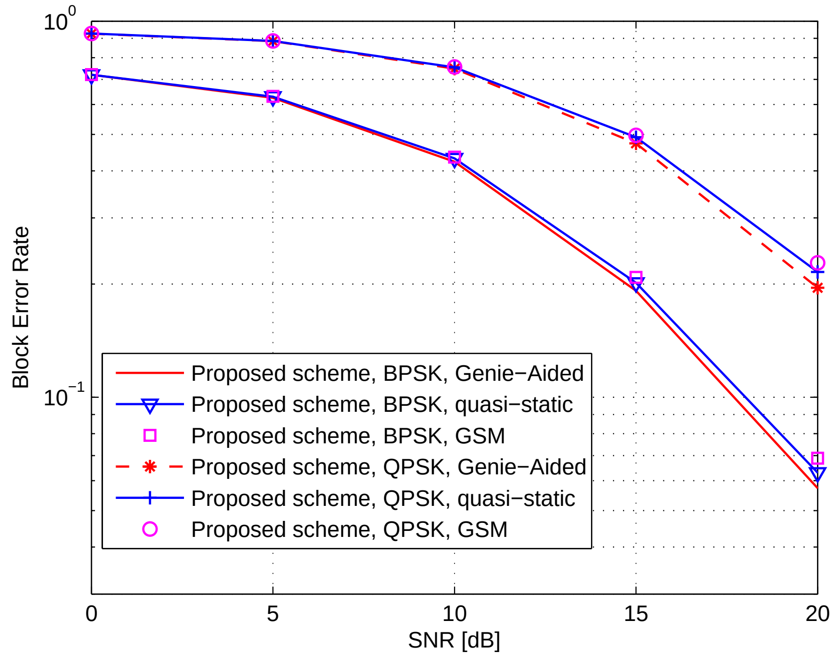

Fig. 3. Simulated BLER performance using Alamouti codes (2 relays) over a GSM channel and quasi-static Rayleigh fading channel.

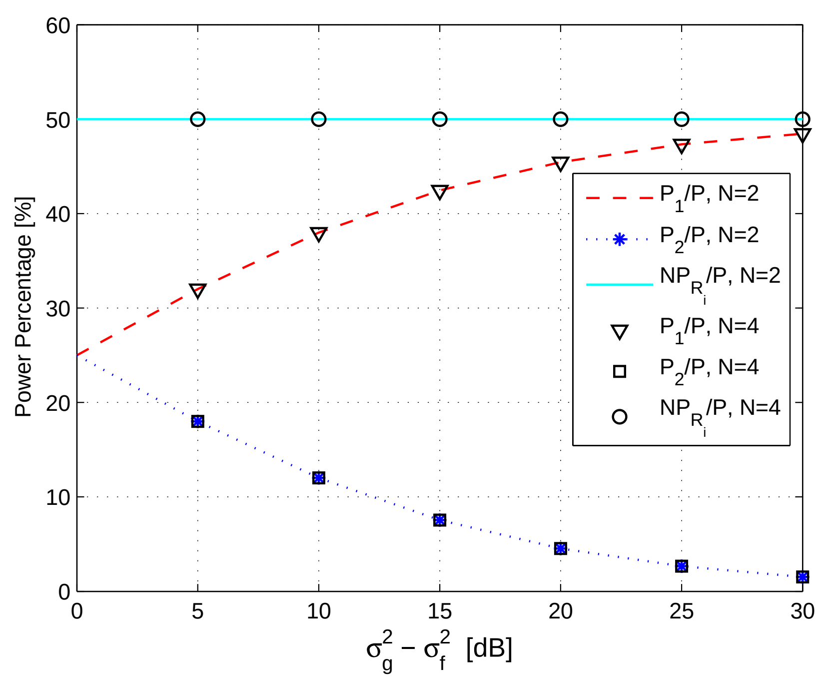

Fig. 4. Optimum power allocation between source and relay nodes.

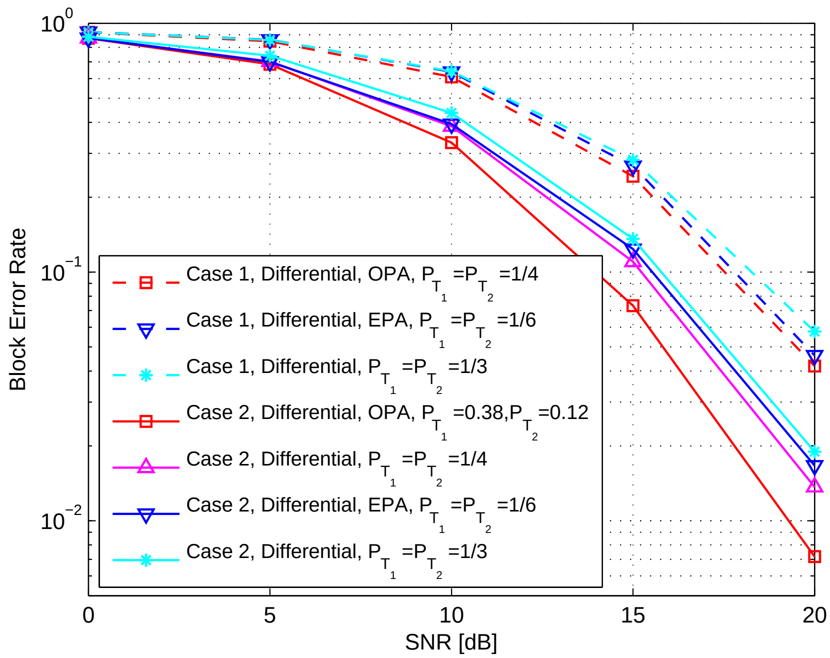

Fig. 5. Simulated BLER performance by the proposed DDSTC-ANC using SORC with transmit power allocation (4 relays).